ON ACTIVE LOADING OF A BAR MADE OF A MATERIAL WITH DELAYING YIELD IN THE CASE OF A NONLINEAR HARDENING DIAGRAM

Yu. P. Gulyaev

On the basis of the model proposed by V. S. Lenskii, an exact solution is obtained for the problem concerned with propagation of nonlinear elastic-plastic loading waves in a semiinfinite bar with delaying yield.

In [1] a model was proposed which, on the basis of a linear hardening diagram, describes the effect of yield delay of a material.

It is possible to generalize this model for a case when the relation $\sigma = \sigma(\varepsilon, t)$ is nonlinear. For this the problem of active loading of a semiinfinite bar is reduced to the construction of the corresponding Riemann wave [2].

Let force $\varphi(t)$ act at the end $x = 0$ of a semiinfinite bar $x \geq 0$, the material of which possesses the effect of yield delay. For the sake of being explicit, we assume that the force is tensile. The $\sigma \sim \varepsilon \sim l$ relation in any section of the bar is taken in the form

$$\sigma = E\varepsilon \qquad (\varepsilon < \varepsilon_s)$$
$$\sigma = E\varepsilon_s(t - x/a_0) + \Phi[\varepsilon - \varepsilon_s(t - x/a_0)] \quad (\varepsilon \geqslant \varepsilon_s), \tag{1}$$

Here ε_S is a decreasing function of time, while the function $\Phi(z)$ satisfies the following conditions:

$$\Phi(0) = 0, \qquad 0 < \Phi'(z) \leqslant E, \qquad \Phi''(z) \leqslant 0. \tag{2}$$

For the strain at the end to be active, the pressure must satisfy the condition [3]

$$\varphi'(t) \geqslant E\varepsilon_s'(t). \tag{3}$$

In the region of elastic deformations (Fig. 1, region 1) the equation of motion has the form

$$\frac{\partial^2 u}{\partial t^2} = a_0^2 \frac{\partial^2 u}{\partial x^2}, \qquad a_0^2 = \frac{E}{\rho}. \tag{4}$$

The solution of Eq. (4) for zero initial data is given by the quadrature

$$u(x, t) = -\frac{a_0}{E} \int_0^{t - x/a_0} \varphi(\xi)\, d\xi$$
$$\varepsilon = \frac{\partial u}{\partial x} = \frac{1}{E}\varphi(t - x/a_0), \quad v = \frac{\partial u}{\partial t} = -\frac{a_0}{E}\varphi(t - x/a_0). \tag{5}$$

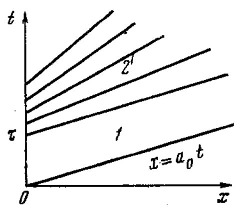

Fig. 1

The determination of the stress-strain state inside the elastic-plastic region 2 reduces to finding the solution of the equation

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$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2} + \frac{a^2 - a_0^2}{a_0} \varepsilon_s' (t - x/a_0), \quad a^2 = \Phi' (\varepsilon - \varepsilon_s) / \rho, \qquad a^2 (0) = a_0^2 = \frac{E}{\rho} \tag{6}$$

for the following boundary conditions:

$$\sigma (0, t) = \varphi (t), \quad u_1 = u_2 \quad \text{for} \quad x = a_0 (t - \tau) \tag{7}$$

Here τ is the instant at which plastic strains occur at the end of the bar.

It is easy to show that the characteristic $x = a_0(t - \tau)$ is a line of weak discontinuity. There the last condition (7) can be replaced by the stipulation of continuity of velocities and strains for $x = a_0(t - \tau)$.

We introduce into the analysis a new function $U(x,t)$, given by the equation

$$U (x, t) = u (x, t) + a_0 \int_{\tau}^{t - x/a_0} \varepsilon_s (\xi) \, d\xi$$
$$\varepsilon^* = \frac{\partial U}{\partial x} = \varepsilon - \varepsilon_s (t - x / a_0), \quad V = \frac{\partial U}{\partial t} = v + a_0 \varepsilon_s (t - x / a_0) . \tag{8}$$

Then Eq. (6) assumes the form

$$\frac{\partial^2 U}{\partial t^2} = a^2 (\varepsilon^*) \frac{\partial^2 U}{\partial x^2} . \tag{9}$$

The function $U(x,t)$ here must satisfy the following conditions:

$$\varepsilon^*|_{x=0} = \varepsilon_0^* (t), \quad \varepsilon^* = V = 0 \quad \text{for} \quad x = a_0 (t - \tau) \tag{10}$$

where $\varepsilon_0^*(t)$ is a given function of time.

Equation (9) constitutes a quasi-linear equation of the hyperbolic type, which is equivalent to the following system of characteristics:

$$dx = \pm a (\varepsilon^*) \, dt, \qquad dV = \pm a (\varepsilon^*) \, d\varepsilon^* . \tag{11}$$

The last conditions (11) can be integrated along the corresponding characteristics in the xt plane

$$V = \pm \int_0^{\varepsilon^*} a (\xi) \, d\xi + c_{1,2} \tag{12}$$

where the constants c_1 and c_2 have, generally speaking, have different values on different characteristics.

We show that in the region 2 the integral of Eq. (9) exists,

$$V = - \int_0^{\varepsilon^*} a (\xi) \, d\xi . \tag{13}$$

Indeed, along the characteristics of negative slope the relation

$$V = - \int_0^{\varepsilon^*} a (\xi) \, d\xi + c_2$$

holds.

Since the characteristics of negative slope intersect the line $x = a_0(t - \tau)$ on which $\varepsilon^* = V = 0$, then $c_2 \equiv 0$. Thus the integral (13) exists in the region 2.

The characteristics of positive slope are straight lines. Indeed, from the existence of the integral (13) and the characteristic condition

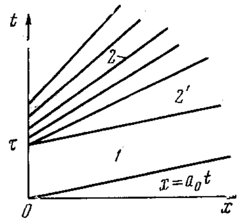

Fig. 2

$$V=\int_0^{\varepsilon^*} a(\xi)\,d\xi + c_1$$

it follows that ε^* and V are constant along each characteristic of positive slope. Consequently, their equation has the form

$$x = a(\varepsilon^*)(t - t_0), \tag{14}$$

Here t_0 is any point on the t axis for which, in accordance with the boundary condition (10), the force $\varphi(t_0)$ or, which is the same thing, $\varepsilon_0^*(t)$ is known.

The solution of the problem in the region 2 is obtained in the form of the Riemann wave [2]

$$\varepsilon^* = \varepsilon_0^*(t - x / a(\varepsilon^*)), \qquad V = -\int_0^{\varepsilon^*} a(\xi)\,d\xi. \tag{15}$$

The strains and velocities of sections of the bar are expressed by the following equations:

$$\begin{gathered} \varepsilon = \varepsilon_0^*(t - x / a(\varepsilon - \varepsilon_s)) + \varepsilon_s(t - x / a_0) \\ v = -\int_0^{\varepsilon - \varepsilon_s} a(\xi)\,d\xi - a_0\varepsilon_s(t - x / a_0). \end{gathered} \tag{16}$$

In particular, if the function $\Phi(z)$ is linear,

$$\Phi(z) = E_1 z \qquad (E_1 < E)$$

then the solution (16) is transformed into the well-known solution for plastic regions given in [1]. From the solution thus obtained we see that elastic relaxation waves, arising as a result of spontaneous reduction of the yield point of the material with time, are imposed on the plastic loading waves in the region 2.

If $\Phi'(0) \neq E$, then a region of pure relaxation of stresses (the region 2' in Fig. 2) appears between the regions 1 and 2, i.e., in this region the solution has the form

$$\varepsilon = \varepsilon_s(t - x / a_0), \qquad v = -a_0\varepsilon_s(t - x / a_0). \tag{17}$$

In the region 2 in this case the solution (16) remains valid.

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